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Economic Design of Reinforced Concrete Columns under Direct Load and Uniaxial Moments

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Abstract: Columns are important structural elements constructed integrally with framing concrete beams and slabs to provide resistance to both axial forces and bending moments. A key aspect in the design of columns is the strength of the column cross-section subjected to combine 'axial force' and 'bending moment'. Designing a reinforced concrete (RC) column to resist an axial load and uniaxial moment is an iterative procedure which involves tedious calculations. The design is affected by many factors such as eccentricity of loading, size of the column cross section, percentage of steel, position of neutral axis, grade of steel, and grade of concrete, thereby requiring the use of interaction diagrams. In the present study, an attempt has been made to determine optimum design of reinforced concrete columns under direct load and uniaxial moments that satisfies all code requirements of IS 456-2000 and also results in minimum cost thereby. The 'percentage of reinforcement' and 'depth of neutral axis' are considered as design variables and using the capacity of swarm intelligence i.e. particle swarm optimization technique based on inertia weight, optimum design of RC column has been achieved. Some design examples are presented to demonstrate usefulness of swarm intelligence algorithm.

Keywords: *Reinforced concrete, Optimum design, Swarm intelligence, Code requirements*

1. Introduction

Typically, columns of a framed building are vertical members subjected not only to compressive loads but also significant bending - due to eccentricity of loads. In such columns the strain distribution across the section is not uniform, but varies linearly across the section as in the case of beams [1]. The load carrying capacity of an eccentrically loaded column depends upon the size of the column, disposition of reinforcement, stress-strain curves of the materials used, and above all, the eccentricity of load. The analysis and design of eccentrically loaded columns is generally cumbersome and may require several hours of computational effort [2]. In order to ease the design process and overcome lengthy calculations, interaction diagrams prepared and published by BIS can be used [3]. The traditional practice involves performing preliminary elastic analysis based on an assumed cross section and checking the member for its adequacy against strength and other codal design requirements [4]. If the codal requirements are not satisfied then the sectional dimensions are modified repeatedly. This iterative procedure is carried on until codal requirements are satisfied, and the process often leads to an un-economical design. Thus, conventionally the designed cost of reinforced concrete structure is highly dependent upon the experience of the structural designer. In order not to make 'optimality' person specific, certain well defined optimum methods/ optimality criteria can be used to carry out safe and optimum column design in reasonable amount of time. Modern artificial intelligence procedures define the structure based on design variables, and automatically design and validate the structure. Thereafter, they redefine it by means of an optimization algorithm that controls the flow of a large number of iterations in search for optimum structure design. Many researchers have performed optimum column design as members of RC frame using genetic algorithm [5-6]. A few studies are also available in which design optimization task simulates swarm behavior [7-9]. In the present study short tied square and rectangular columns have been optimized using swarm intelligence. A C++ program has been developed for optimum column design and also to capture the design points, i.e. the axial and moment resisting capacity of the column for various cover to depth ratio, grade of steel and area of reinforcement.

2. Optimization Technique

2.1. Particle Swarm Optimization (PSO)

Particle swarm optimization is a population based stochastic optimization method (Kennedy and Eberhart, 1995) [10]. It explores the optimal solution from a population swarm of moving particle vectors, based on a fitness function. Each i^{th} particle vector represents a potential solution and has a position $x_i(t)$ and a velocity $v_i(t)$ at time t in the problem space. Each i^{th} vector keeps a record of its individual best position $P_i(t)$, which is associated with its own best fitness achieved so far, at any time in the iteration process. This value is known as 'pbest'. Moreover, the optimum position among all the particles obtained so far in the swarm is stored as the global best position $P_g(t)$. This location is called 'gbest'. The new velocity of particle is updated as follows:

$$v_i(t+1) = w(t)\,v_i(t) + c_1 r_1 \left(P_i(t) - x_i(t)\right) + c_2 r_2 \left(P_g(t) - x_i(t)\right) \quad (1)$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (2)$$

Thus, $v_i(t)$ is the velocity of particle i at time t and $x_i(t)$ is the position of particle i at time t. $w(t)$ is inertia weight at time t in the first part and represents the memory of a particle during search. The inertia weighting function at each iteration is given as:

$$w(t) = w_{max} - (w_{max} - w_{min}) \times itr / itr_{max} \quad (3)$$

w_{max} and w_{min} are the maximum and minimum values of w inertia weight respectively, itr_{max} is the maximum number of iterations and itr is the current iteration number. The first right hand term in (1) enables each particle to perform a global search by exploring a new search space, whereas the last two terms represent cognitive and social parts respectively in which c_1 and c_2 are positive numbers illustrating the weights of the acceleration terms that guide each particle toward the individual best and the swarm best positions respectively. r_1 and r_2 are uniformly distributed random numbers in (0, 1) and N is the number of particles in the swarm. Each particle changes its position based on the updated velocity according to the equation (2) which is known as flight formula. In this way, 'velocity updating' (1) and 'flight formula' (2) help the particles to locate an optimal solution in the search space. In order to keep the particles from moving too far beyond the search space, their velocities have been clamped by limiting the maximum velocity of each particle. Most of the time, value of maximum velocity is selected empirically, according to the characteristics of the problem. It is important to note that if the value of this parameter is too high, then the particles may move erratically, going beyond a good solution; on the other hand, if is too small, then the particle's movement is limited and the optimal solution may not be reached.

2.2. Column Design Method

A column subjected to varying magnitudes of 'P' and 'M' will act with its neutral axis at different locations as described in [4]

Case 1: When P = P_u and M=0 (Axial load only)
Case 2: When P = 0 and M = M_u (Moment only)
Case 3: When both P and M have non-zero values.

For every load P there is a particular value of M which will cause failure. Thus there will be infinite combinations of P_u and M_u which can safely act together for a given RC section. The particular value of M_u for a given value of P_u can be found only by trial and error and the work is quite tedious. It will be more convenient especially for routine design to construct a curve showing the P_u-M_u combinations (P-M interaction curve). This can be made non-dimensional by using a diagram such as shown in Fig. 1.

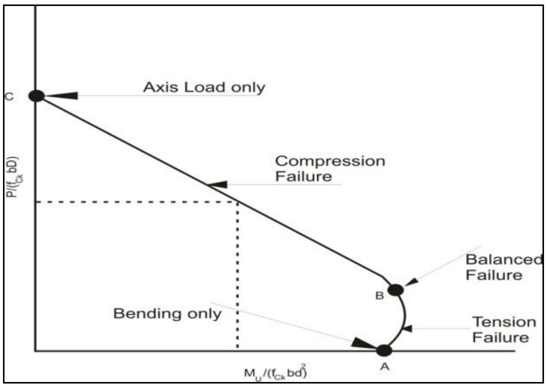

Figure 1 *P-M interaction diagram for column*

Explanation of P-M interaction diagram

Compression Failure- Section is subjected to axial load only. In this case strain distribution at failure is uniform and equal to 0.002. Neutral axis lies at infinity. In P-M diagram, points B to C represents compression failure condition where failure is initiated by concrete reaching its ultimate strain first.

Tension Failure- In this case NA will be inside the section. Section is subjected to pure bending. Means NA is at k_1Dc and k_1 is 1 in this case. The points from A to B on the interaction diagram represent tension failure condition, where steel on tension face reaches yield point before failure.

Balanced Failure- Balanced failure condition is where concrete reaches failure strain and steel reaches the yield strain in tension simultaneously. In the first case, NA will lie between the balanced and axial load condition and in the second case it will lie between pure bending and balanced failure conditions. In this case the NA will be within the section and its position can be at strain in concrete equal to 0.0035 and strain in steel equal to yield strain.

When the NA is outside the section, part of the parabola will be outside the concrete area. The values of kDc will be greater than Dc.

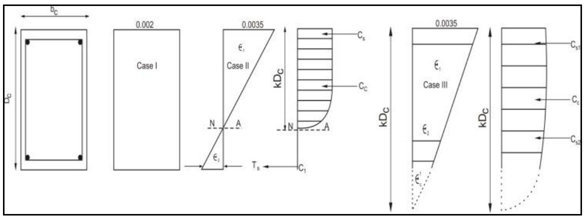

Figure 2 *Strain distributions in column section*

3. Column Optimization Problem Formulation

3.1. Objective Function

The cost of reinforced concrete column element includes

$$C = C_{st} V_{st} + C_c V_c \quad (4)$$

C is the total cost of column; C_{st} cost of steel per unit volume of steel; V_{st} total volume of steel ; C_C cost of concrete per unit volume of concrete; V_C total volume of concrete in the column.

Dividing equation (4) by C_C

$$\frac{C}{C_C} = \frac{C_{st}}{C_C} V_{st} + V_C$$

and substituting $\frac{C}{C_C} = Z$, $\frac{C_{st}}{C_C} = \alpha$ (cost ratio) and $V_C = V_G - V_{st}$, equation (5) is obtained. V_G is the gross volume of structural element.

Since C_C is a constant parameter for a given place, the objective function Z represents total cost of frame component required to minimize.

$$\text{Minimize } Z = (\alpha - 1)V_{st} + V_G \quad (5)$$

Volume of steel V_{st} depends upon area of steel and its provided length. Similarly gross volume of concrete depends upon cross sectional area and length of column.

3.2. Design Variables and Constraints for Column Optimization

Without violating any of the constraints stated below, column optimization consists in determination of depth D_C and width b_C, with percentage area of longitudinal reinforcement p and ratio of depth of neutral axis to depth of column k as independent design variables such that the cost of column is minimized.

Condition for axial load capacity of the column

The axial load carrying capacity of the column shall be greater than the actual load applied on it.

$$0.36\, f_{ck}\, b_C\, k\, D_C + \sum_{i=1}^{n} (f_{si} - f_{ci}) \frac{p_i\, b_C\, D_C}{100} \geq P$$

Condition for moment capacity of the column

The moment carrying capacity of the column shall be greater than the actual moment applied on it.

Condition for minimum and maximum longitudinal reinforcement in the column

As per IS 456: 2000, the cross-sectional area of longitudinal reinforcement shall not be less than 0.8 percent of the gross cross-sectional area of the column.

$$p \geq 0.8 \quad \text{and} \quad p \leq 4.0$$

Condition for minimum number of longitudinal bars

The number of longitudinal bars provided in a column shall not be less than 4.

$$\frac{total\ area\ of\ long.\ re\inf}{area\ of\ one\ bar} \geq 4$$

Condition for maximum peripheral distance between longitudinal bars

The spacing of longitudinal bars measured along the periphery of column shall not be more than 300 mm. Mathematically, it can be expressed as

$$d_p \leq 300$$

Condition for minimum width of column

From practical point of view, the width of column shall be greater than the width of beams attached to it. As per IS 13920:1993 [11], minimum width of column is kept as 300 mm.

4. Results and Discussion

Since Particle swarm optimization is applicable to unconstrained optimization problems, therefore above constrained optimization problem is transformed to an unconstrained optimization one for the application of current optimization technique. The previously defined constraints are normalized and violations of constraints are incorporated using exterior penalty function method to constitute the unconstrained objective function. The independent design variables as given in previous section are searched from defined search space. All columns have been designed according to limit state method as per IS456:2000 and then a set of solution is obtained by applying PSO. Constant parameter values of the algorithm that were found in tune with the variables are as follows:

$$c_1 = c_2 = 2,\ w_{max} = 0.9,\ w_{min} = 0.4,\ v_{max} = 4$$

The population size and maximum number of iterations (fixed parameters) are taken as 20 and 500 respectively for the applied algorithm. It is necessary to define the upper and lower bounds of design variables of column design problem for the random selection of the population.

The design procedures is developed in a generalized form which accepts different parametric values related to column design such as its effective length, axial load, moment acting on it and material properties. All optimization runs have been carried out on a standard PC with a Intel® Core™ i3 CPU M350 @2.27 GHz frequency and 3 GB RAM memory. The algorithm has been coded in Turbo C++ installed in Windows 7 (32 bit operating system). All the columns are considered as uniaxial columns loaded with a factored axial force and factored bending moment. The design of short columns is dependent on the stresses in the reinforcing steel. These stresses are used to generate the strength load – moment interaction diagram for the column. Feasible solutions are generated based on the restrictions and specifications outlined in IS 456:2000 [4]. The design variables for column design are percentage of reinforcement and depth of neutral axis. The design of columns is based on strength of the column cross-section subjected to combined axial force and bending moment. Considering all the possible load and moment combinations for a given cross section, a computer aided design program has been developed for calculating the strength of a column. As an example of optimum design using PSO, strength of a column is designed for given axial load of 960 kN and uniaxial moment of 250 kN-m. The minimum dimension of the column shall not be less than 300 mm. Cover ratio and minimum column depth to width ratio have been taken as 0.1 and 1.0 respectively.

Area of longitudinal steel (as percentage of column area), ‘ p ’ lies between 0.8% to 4.0%. The grades of concrete and steel have been taken as M25 and Fe415 respectively. An unsupported length of column is considered as 3.0 m. Also, effective length ratio for the columns has been taken as 1.2 and cost ratio (α) as 100. For these given set of input values, output design parameters are the cross-sectional dimensions of column - 780 mm depth and 300 mm width. Optimum area of longitudinal reinforcement obtained was 0.8% which required 266 iterations of PSO. The time taken for optimum design of column was 2 seconds. The generalized cross sectional view of column in which number of longitudinal bars may vary as per the design, has been illustrated in Fig. 3.

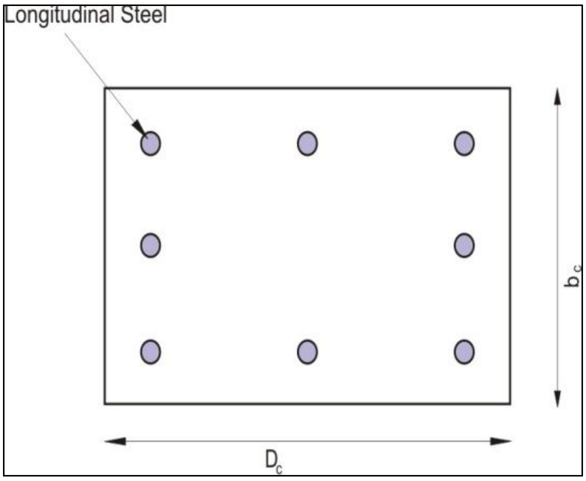

Figure 3 *Generalized sectional view of column*

The PSO convergence history or progress of design improvements for the above defined problem is shown in Fig. 4. It has been observed that optimum design is achieved in few initial iterations with given swarm size and after that there is no significant improvement. Maximum five runs of optimum column design program has been made and standard deviation obtained is found to be negligible which proved robustness of the algorithm.

Some optimum design results for varying axial loads and moments are shown in Table 1. For the given set of input parameters, the optimum design variables viz. percentage of longitudinal steel, cross sectional dimensions have been presented. Number of iterations required for optimization has also been indicated in the same table.

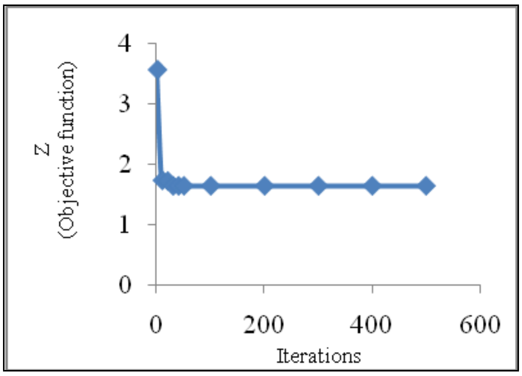

Figure 4 *Progress of design improvements*

Table 1: *Optimum design results of five columns subjected to different axial loads and moments*

Column Example	Axial load (kN)	Moment (kN-m)	Depth (mm)	Width (mm)	Steel (%)	Z_{Total} (C/C$_c$)	No. of Iterations
C_1	1100	230	780	300	0.8	1.637	276
C_2	309	217	780	300	0.8	1.650	310
C_3	283	85	470	300	0.8	0.999	273
C_4	245	65	410	300	0.8	0.881	281
C_5	100	32	310	300	0.8	0.677	336

5. Conclusions

The following conclusions are drawn based on the present study:

1) The optimization problem shall be formulated as a design problem rather than analysis problem, in such a manner that traditional design procedure is imitated.
2) The optimum designs are achieved at minimum percentage of steel and minimum width of column specified by the designer.
3) The proposed technique has a very few parameters to define at initial stage and thus is easier to apply than other traditional optimization techniques.
4) It has been viewed that reduction in both steel area as well as concrete volume contributes towards optimization of reinforced concrete columns.
5) Optimum design results have been achieved in few iterations and time taken to carry out optimization procedure is very less.

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