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Multivariable Optimization of Isotropic Cylindrical Shells

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Abstract: Structural optimization plays an essential role in the structural design process because it represents a systematic method to improve the designs with respect to certain design criteria. The present work aims at improving the static performances of cylindrical shells by adopting structural design optimization process which leads to lighter structures with minimum cost. Here, the Structural Optimization (SO) for shells are carried out using gradient based Sequential Quadratic Programming (SQP) method. The structural analysis is carried out with Finite Element (FE) method using a four noded shell element based on degenerated shell theory. With elementwise thickness as the design variables, the optimal solution is found by the use of structural optimization algorithm which integrates the finite element method and sequential quadratic programming method. The optimization problem is formulated as single objective to minimize the weight of the shell with maximum displacement as constraints. Numerical studies are carried out to get the optimal thickness for shells under different boundary condition. The effect of element wise thickness variation on the optimum design of shells is presented.

Keywords: *Structural Optimization, Finite Element method, Single objective, Multivariable, Sequential Quadratic Programming*

1. Introduction

Structural design process using mathematical models is extensively used in civil, automotive and aircraft industries for the design and development of new structures or to improve the performance of the existing structures. In a structural optimization process, the objective of the designers is to find the best design in terms of cost, weight, reliability, mechanical properties or aesthetics etc. A key point of the design process is to translate these objectives into criteria expressed mathematically to measure if a certain design is satisfactory or not.

The objective function, also called the cost function is the function to be minimized (or maximized) during the optimization process and constitutes the criteria by which a certain design is chosen among a group of alternatives. Design variables are numbers whose value can be freely varied by the designer to define a designed object. Therefore structural configuration is a function of the chosen design variables and these are actually the unknowns of the optimization problem. Constraints are the restrictions to the problem that are to be satisfied in order to have an acceptable design. That is, these constraints define feasible domain.

In order to get the best design solution, many gradient-based optimization algorithms are available, which requires the function values and sensitivity information at given design variables. For a given design variable that defines the structural model, structural analysis provides the values of the cost and constraint functions for the algorithm. Generally, finite element based structural analysis is used to numerically evaluate the values of the cost and constraint functions. Further, design sensitivities of the cost and constraint functions must also be supplied to the optimization algorithm. Then, the optimization algorithms calculate the best possible design of the problem. The performance of an optimization algorithm critically depends on the characteristics of the design problem and the types of cost and constraint functions.

2. Literature Review

In this section, a brief literature survey is done based on the topic chosen for the present work. First, the development and application of the finite element methods, as it pertains to structural analysis is discussed. Next, reviews on various methods used for the structural design sensitivity analysis for static responses of structure are reviewed. Finally, the development and application of optimization methods to structural design optimization is discussed.

2.1. Finite Element Method

Finite element procedures are extensively used in the analysis of structures, heat transfer and fluid mechanics. A lot of research has been done towards the technique and a large number of publications on finite element method are now available.

Cheung and Au [1] presented the isoparametric spline finite strip method for degenerate shells. In the formulation both the geometry and the displacement fields are represented by uniform cubic B-spline curves. Linear, quadratic and cubic finite strips were used for the static analysis of shells. Accuracy and performance was evaluated for different shells with different support conditions. Ashwell and Sabir [2] developed a new cylindrical shell finite element based on simple independent strain functions. The rectangular cylindrical shell element had 20 dofs and satisfied the conditions for rigid body displacements and constant strains. The method was applied to various shells with different support conditions.

Djoudi and Bahai [3] developed a cylindrical strain based shallow shell finite element for linear and nonlinear analysis of cylindrical shells. The element was rectangular in plan having nodes at the corners and each node having five degrees of freedom. The suitability of the element was also tested for natural frequencies of cylindrical shells. Liu etal. [4] carried out static and free vibration of general shell structures using element free Galerkin method. A discrete singularity-free mapping between the five and six degrees of freedom was constructed by exploiting the geometry connection between orthogonal group and unit sphere. The phenomenon of shear and membrane locking were illustrated by showing the membrane and shear energy as a fraction of total energy. Afonso and Hinton [5] carried out free vibration analysis for plate and shell structures. The shape and thickness distribution of the structure were represented using Coons Patches using a nine noded, degenerated Huang Hinton element with assumed strain fields. Its performance was evaluated for shells with arbitrary thickness variation and boundary conditions.

2.2. Sensitivity Analysis

Sensitivity analysis has evolved over the past four decades and has been found to be useful in many engineering applications. In the early stages, sensitivity analysis was used to calculate the effect of changes in design variables of analytical models. Haftka and Adelman [6] surveyed the methods applicable to the calculation of structural sensitivity derivatives for finite element modeled structures and discussed on four main topics: derivatives of static response (displacements and stresses), eigen values and eigen vectors, transient response and derivatives of optimum structural designs with respect to problem parameters. Yamazaki and Vanderplats [7] carried out design sensitivity analysis by the direct differentiation method for isoparametric curved shell elements. Sensitivity parameters included geometric variables which influence the size and shape of the structure as well as shell thickness. The method was applied for the sensitivity calculations of displacement, stresses, buckling load and natural frequency of cylindrical shells. Zhang and Domaszewski [8] developed a new efficient sensitivity procedure for the optimization of shell structures without access to the finite element source code. The implementation was performed based on ABAQUS code. Kirchhoff flat shell elements were taken into account in the study with the element thickness as the design variable. Afonso and Hinton [5] used an automated approach to carry out sensitivity analysis. Design variables that specify either the shape or thickness distribution of the structures were considered. Special attention was focused on the sensitivity calculations and problems connected with their accuracy and performances were highlighted when the semi-analytical and finite difference methods were used.

2.3. Optimization

Optimization techniques have been used to solve real-life design problems for more than six decades. A large number of studies have been done on the shape, topology and structural optimization of various shells [9, 10, 11, 12, 13, 20, 21].

Camprubiet.al. [14] discussed the significance of a reliable finite element formulation in the optimization process. The optimization process and analytical sensitivity was carried out based on the Discrete Shear Gap (DSG) method. The effect of shell locking on shell optimization was also studied using a standard displacement shell formulation and DSG method. Hinton and Rao [15] carried out structural shape optimization of shells using two noded Mindlin-Reissner finite strips. The thickness and shape variables that define the cross section of the structure were considered as design variables.

The objective was to minimize the strain energy with the total volume as constraint. It was observed that the minimization of strain energy lead to optimum structures in which the deflection and stress resultant was considerably reduced. Afonso and Hinton [5] made an automated approach to carry out sensitivity analysis to obtain optimum shapes for plates and shells in which the natural frequencies were maximized. Sensitivity analysis was carried out using semi-analytical and finite difference method. The optimal solution was found by the use of structural optimization algorithm which integrates the finite element module, sensitivity analysis and a sequential programming algorithm. Optimal forms were obtained for a set of benchmark problems using the two sensitivity analysis techniques and their results were compared.

3. Objective

The objective of the present work is to investigate the single objective optimization of isotropic cylindrical shell structures considering static performance measures under different boundary conditions. The finite element formulation of isotropic cylindrical shells is based on degenerated shell theory. An algorithm is developed to study the single objective optimization of shells, by minimizing its weight, under displacement constraints. An optimized thickness of shell is ascertained, considering element wise thickness distribution as design variable using sequential quadratic programming method.

4. Shell degenerated from 3D solid

Figure 1 shows a three dimensional element having curved edges at the top and bottom surfaces while a straight edge in the thickness direction, which also shows the degeneration from three dimensional to two dimensional one [22].

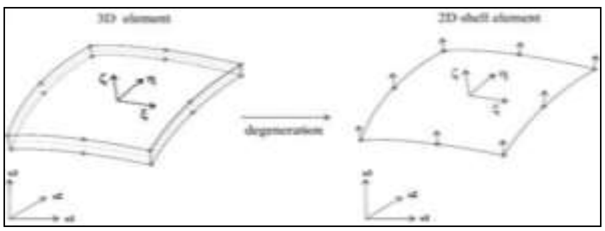

***Figure 1** Degeneration of a three dimensional shell element into a two dimensional element*

A curvilinear coordinate system with (ξ, η, ζ) with origin in the mid-plane of the shell element is defined. ξ and η are the curvilinear coordinates in the mid plane which varies from -1 to 1. ζ is the linear coordinate in the thickness direction which also varies from -1 to 1. A generic point of a shell may now be described in terms of curvilinear coordinates as

$$x_i(\xi,\eta,\zeta) = \sum_{k=1}^{n} N_k(\xi,\eta)x_i^k + \sum_{k=1}^{n} N_k(\xi,\eta)H^k(\zeta)V_{3i}^k \quad (i=1,2,3) \quad (1)$$

Where x_i^k is the position vector of node k in the middle plane. V_{3i}^k is the unit vector at the node k and n is the number of nodes per element. N_k is the two dimensional shape function in the curvilinear coordinate and H^k is the one dimensional shape function in the linear coordinate. The unit vector V_{3i}^k is defined as

$$V_{3i}^k = \frac{(x_i^k)^{top} - (x_i^k)^{bottom}}{\| (x_i^k)^{top} - (x_i^k)^{bottom} \|} \quad (2)$$

Where *top* and *bottom* indicate the top and bottom surfaces of the shell and ‖ ‖ denotes the Euclidean norm.

4.1. Displacement field of the shell

The displacement field of the shell can be written as

$$u_i(\xi,\eta,\zeta) = \sum_{k=1}^{n} N_k(\xi,\eta)u_i^k + \sum_{k=1}^{n} N_k(\xi,\eta)H^k(\zeta)\left(-V_{2i}^k\theta_1^k + V_{1i}^k\theta_2^k\right) \; (i=1,2,3) \quad (3)$$

In which u_i is the displacement along the x_i axis u_i^k is the nodal displacement at the node k and unit vectors V_{1i}^k and V_{2i}^k lie along the reference surface. V_{1i}^k, V_{2i}^k, V_{3i}^k are perpendicular to one another. θ_1^k and θ_2^k are rotational degrees of freedom along the unit vectors V_{1i}^k and V_{2i}^k respectively. Figure 2 shows unit vectors to the coordinates at mid plane of the shell element.

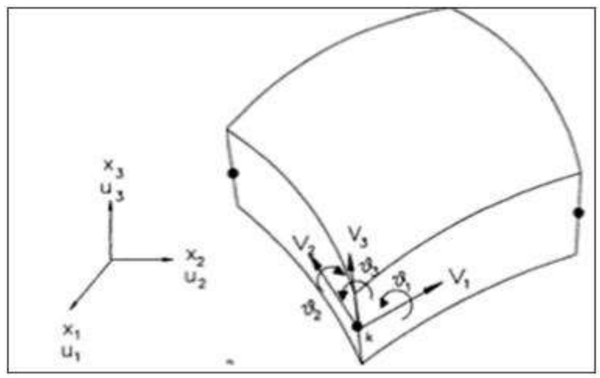

***Figure 2** Shell element showing unit vectors at mid plane of the shell*

4.2. Strain Displacement Relations

Six strain components are computed from Eq. (3) by taking the derivative with respect to x_i axis. The result can be written in matrix form as,

$$\{\varepsilon\} = [B]\{d\} \quad (4)$$

Where

$$\{\varepsilon\} = \{\varepsilon_{11}\varepsilon_{22}\varepsilon_{33}\gamma_{12}\gamma_{23}\gamma_{13}\}^T \quad (5)$$

$$[B] = [B^1 B^2 \dots B^n] \quad (6)$$

$$\text{and } \{d\} = \{d^1 d^2 \dots d^n\} \quad (7)$$

The detailed expression for $[B^k]$ can be found in [18].

4.3. Jacobian Matrix

In order to compute the derivatives such as $\frac{\partial N^k}{\partial x_i}$ and $\frac{\partial H^k}{\partial x_i}$ the jacobian matrix is required & is defined as

$$[J] = \begin{bmatrix} x_{1,\xi} & x_{2,\xi} & x_{3,\xi} \\ x_{1,\eta} & x_{2,\eta} & x_{3,\eta} \\ x_{1,\zeta} & x_{2,\zeta} & x_{3,\zeta} \end{bmatrix} \quad (8)$$

Where

$$\frac{\partial x_i}{\partial \xi} = \sum_{k=1}^{n} \frac{\partial N^k}{\partial \xi} x_i^k + \frac{\partial N^k}{\partial \xi} H^k V_{3i}^k \quad (9)$$

$$\frac{\partial x_i}{\partial \eta} = \sum_{k=1}^{n} \frac{\partial N^k}{\partial \eta} x_i^k + \frac{\partial N^k}{\partial \eta} H^k V_{3i}^k \quad (10)$$

$$\frac{\partial x_i}{\partial \zeta} = \sum_{k=1}^{n} N^k \frac{\partial N^k}{\partial \zeta} V_{3i}^k \quad (11)$$

The inverse of the matrix is called $[R]$, then

$$\frac{\partial N^k}{\partial x_i} = R_{i1} \frac{\partial N^k}{\partial \xi} + R_{i2} \frac{\partial N^k}{\partial \eta} (i = 1,2,3) \quad (12)$$

$$\frac{\partial H^k}{\partial x_i} = R_{i3} \frac{\partial H^k}{\partial \zeta} (i = 1,2,3) \quad (13)$$

4.4. Constitutive Equations

Stresses and strains can be expressed as

$$\{\sigma'\} = [D']\{\varepsilon'\} \quad (14)$$

Where $\{\sigma'\}$ and $\{\varepsilon'\}$ are the stresses and strain components in the local axes which are set along the reference plane made of vectors V_1 and V_2 as shown in Fig. 2. The constitutive matrix $[D']$ for an isotropic material is given as:

$$[D'] = \begin{bmatrix} \frac{E}{(1-\upsilon^2)} & \frac{\upsilon E}{(1-\upsilon^2)} & 0 & 0 & 0 & 0 \\ \frac{\upsilon E}{(1-\upsilon^2)} & \frac{E}{(1-\upsilon^2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\upsilon)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{mE}{2(1+\upsilon)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{mE}{2(1+\upsilon)} \end{bmatrix} \quad (15)$$

In which E and υ are the elastic and Poisson's ratio, respectively. This matrix includes the transverse shear deformation. The fifth and sixth rows are for the transverse shear deformation. m is the shear correction factor which is chosen as 5/6 for an isotropic material.

The material property matrix $[D']$ is transformed into a matrix in terms of the stresses and strains of the global axes. The transformed material property matrix is expressed as [18]:

$$[D] = [T_l]^T[D'] [T_l] \quad (16)$$

Where $[T_l]$ is the transformation matrix.

4.5. Element Stiffness Matrix

The element stiffness matrix is computed from

$$[K] = \int_{\Omega} [B]^T[D][B]\, d\Omega \quad (17)$$

However the rotational degrees of freedom are expressed in terms of the local vectors and they should be expressed in terms of the global axes so that they can be assembled properly. Such a transformation is obtained from

$$\{d^k\}_{global} = \{T_{rot}\}\{d^k\} \quad (18)$$

Where $\{T_{rot}\}$ is as given in [18].

For a four node shell element, the transformation matrix for rotational degrees of freedom becomes

$$[\overline{T_{rot}}] = \begin{bmatrix} T_{rot} & 0 & 0 & 0 \\ 0 & T_{rot} & 0 & 0 \\ 0 & 0 & T_{rot} & 0 \\ 0 & 0 & 0 & T_{rot} \end{bmatrix} \quad (19)$$

The local degrees of freedom in vector in Eq. (18) includes θ_3 for the proper coordinate transformation as shown below

$$\{d_k\} = \{u_1 \quad u_2 \quad u_3 \quad \theta_1 \quad \theta_2 \quad \theta_3\} \quad (20)$$

As a result element stiffness matrix $[K]$ should be expanded to incorporate the degree of freedom at each node. Thus, the transformed element stiffness matrix is expressed as

$$[\bar{K}] = [\overline{T_{rot}}]^T [K] [\overline{T_{rot}}] \quad (21)$$

5. Static Analysis of Shells

The strain energy of the complete structure, composed of finite elements, is given as

$$U = \frac{1}{2}\{d\}^T[K] \{d\} \quad (22)$$

The external work done by the transverse load on the whole structure is given as

$$W = \{d\}^T[F] \quad (23)$$

Where $\{F\}$ is the global force vector.

Now, the total potential energy of the structure can be expressed as

$$\pi = U - W \quad (24)$$

If the components of the displacement vector $\{d\}$ are $\{d\}^T = [q_1 \quad q_2 \cdots q_N]$ then the static equilibrium configuration of the structures can be obtained by minimizing the potential energy as:

$$\frac{\partial \Pi}{\partial q_i} = 0 \quad (25)$$

After applying the principle of variational calculus, the basic equation for the static analysis is obtained as

$$[K]\{d\} = [F] \quad (26)$$

The above equation is solved for the unknown displacements $\{d\}$.

6. Single Objective Optimization

The shell optimization problem is formulated as a constrained single objective function with multi design variable i.e. element wise thickness of shell. This is done to study the optimum thickness distribution of shells.

$$\text{Minimize} \quad f(h_i) = \rho \sum_{i=0}^{ne} A_{si} h_i$$

$$\text{Subject to} \quad \delta_{max} \leq \delta_{lim} \quad (27)$$
$$\text{i.e.,} \quad g(h_i) = \delta_{max} - \delta_{li} \leq 0$$
$$h^L \leq h_i \leq h^U$$

Where $f(h_i)$ *is the* objective function and A_{si} and h_i are the area and thickness of i^{th} element of the shell respectively. h_L and h_U are the lower and upper limit of the design variable. δ_{max} is the maximum nodal displacement of shell and δ_{lim} is the limiting value for displacement constraint. *ne* denotes the number of elements and ρ is the mass density of the shell material.

As the number of design variables increase, the optimization procedure becomes more complicated since the algorithm has to act upon a multi-dimensional design space. There are many number of gradient based optimization algorithms available in the literature, which can deal with multi-dimensional, constrained design space. In the present work, Sequential Quadratic Programming Method is employed for the single and multivariable design optimization of shell, because the method has following advantages: the starting point can be infeasible, gradients of only active constraints are needed, and equality constraints can be handled in addition to the inequalities. On account of these factors, in this work a MATLAB optimization tool called "fmincon" based on SQP is employed.

7. Sequential Quadratic Programming

The sequential quadratic programming (SQP) method effectively solves constraint optimization problems using gradient information of objective and constraint functions. In the following section, optimization algorithm using the SQP method is described to find the thickness design vector value for problems given by Eq. (27). The formulation for SQP is based on Newton's method and Karush-Kuhn-Tucker (KKT) optimality conditions for constrained problems. In this method, a quadratic programming (QP) sub problem is constructed from the initial non-linear programming (NLP) program.

A local optimizer is then found by solving a sequence of these QP sub problems using a quadratic approximation of the objective function. Each sub problem has the following form:

$$\text{Minimize} \quad \frac{1}{2} d_k^T H_k d + \nabla f(h_k)^T d_k$$
$$\text{subject to} \quad \nabla g_i(h_k)^T d + g_i(h_k) \leq 0 \quad (28)$$
$$h^L \leq d_k + h_k \leq h^U$$

Where k is the current iteration number, ∇f and ∇g are the gradients of the objective and inequality constraints respectively evaluated at h_k. H_k is the Hessian matrix of the Lagrange function at the current design point h_k, with the current estimate of the Lagrangian multipliers λ_k, and d_k is the vector of design variables in this sub problem representing the search direction to be defined in the original optimization problem. The solution of the QP sub problem produces an estimate of the Lagrange multiplier λ, and a search direction vector d at iteration k, which is used to form a new and improved design point as:

$$h_{k+1} = h_k + \alpha_k d_k \quad (29)$$

The step length parameter α_k is determined through a one dimensional minimization in order to produce a sufficient decrease in the merit function. At the end of the one dimensional minimization, Hessian of Lagrangian, required for the next QP problem is updated using BFGS formula [19]

$$H_{k+1} = H_k - \frac{H_k P_i P_i^T H_k}{P_i^T H_k P_i} - \frac{\gamma \gamma^T}{P_i^T P_i}$$
$$P_i = h_{k+1} - h_k$$
$$\gamma = \theta Q_i + (1-\theta) H_k P_i$$
$$Q_i = \nabla L(h_{k+1}, \lambda_{k+1}) - \nabla l(h_k, \lambda_k) \quad (30)$$
$$\theta = \begin{cases} 1 & \text{if } P_i^T Q_i \geq 0.2 P_i^T H_k P_i \\ \dfrac{0.8 P_i^T H_k P_i}{P_i^T H_k P_i - P_i^T Q_i} & \text{if } P_i^T Q_i < 0.2 P_i^T H_k P_i \end{cases}$$

From the above equation, it can be observed that gradient of Lagrangian is required at the start of each iteration. Lagrangian gradient consists of both objective and constraint function. Therefore, the correct evaluation of the gradient of the objective and constraint function is necessary for convergence of the solution. An efficient semi-analytical design sensitivity analysis technique has been implemented for evaluating the gradients of objective function and behaviour constraint.

8. Validation of Static Deflection of Shell

A standard problem for testing the cylindrical shell finite element is the pinched cylinder shown in Fig 3. A cylinder is pinched across its diagonal direction with a force of 444.8222 N at the centre lengthwise. It has a radius of 0.127 m, length of 0.2629 m, and thickness of $2.3876 * 10^{-3}$ m.

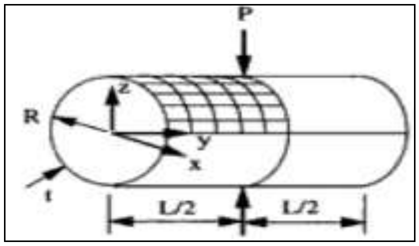

Figure 3 *Pinched Cylinder*

The material has elastic modulus of $72.39*10^9$ N/m^2 and Poisson's ratio of 0.3125. Table 1 shows the comparison of deflection under the load with the analytical solution given in [17] with the finer meshes.

Table 1: *Deflection of pinched cylinder under the load*

Static deflection under the load (in 10^{-3} m)		
FE Mesh	**Present FE solution**	**Analytical solution [17]**
8 X 8	2.6413	2.8931
16 X 16	2.8956	
20 X 20	2.9362	

The barrel vault has been used frequently by many authors as a benchmark for testing shell elements performance. The vault has a radius of 7.620 *m*, a length of 15.240 *m* and a thickness of 0.0762 *m*. The angle subtended is 80°, Young's modulus is 20.68×10^9 N/m^2 and υ = 0. The load acting on the shell is uniform gravity loading of 309.25 N/m^2. The curved edges are supported by diaphragms and the straight edges are free. Figure 4 shows a general shell element and its coordinate system. Table 2 shows the comparison of deflection at the centre of free edge (point C) with the analytical solution given in [16] with the finer meshes.

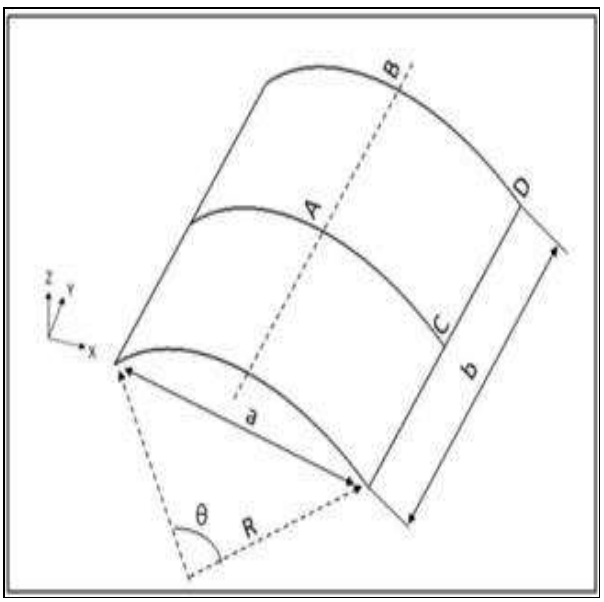

Figure 4.*A typical shell geometry and its coordinate system*

Table 2: *Deflection of barrel vault roof at point C*

Static deflection at point C (in 10^{-2} m)		
FE Mesh	**Present FE solution**	**Analytical solution [16]**
8 X 8	12.8417	9.4063
16 X 16	10.4760	
20 X 20	10.3170	

9. Static Analysis of Square Shell

In this section static analysis of singly curved cylindrical square shells are carried out for with aspect ratio 1 *i.e.*, *a/b*=1, where *a*=width of the shell along the curvature and *b*=length of the shell along the free edges as shown in Fig. 4. The boundary conditions adopted here are all edges clamped (CCCC), curved edges simply supported and straight edges free (SFSF), and cantilever (CFFF). The dimension and material properties of the shell are listed below in Table 3. The shell is analysed with 8x8 mesh.

Table 3: *Dimension and material properties of the square shell*

Radius of the shell R	0.762*m*
Length of the shell b	0.1015322*m*
Angle of inclination θ	7.64˚
Young's Modulus E	$68.948*10^9$ N/m^2
Poisson's Ratio υ	0.33
Thickness of the Shell *h*	$3.3*10^{-4}$*m*
Weight density *ρ*	2657.3 kg/m^3

Table 4: *Maximum deflection of square shell*

Boundary condition	Uniform distributed load (N/m^2)	Maximum deflection (in $10^{-3}m$)
CCCC	8500	0.3787 (at A)
SFSF	250	0.3782 (at C)
CFFF	75	0.3745 (at D)

10. Single Objective Optimization of Square Shell

In this section, numerical examples for single objective optimization of isotropic square shells are presented to demonstrate the formulations and framework developed in the previous sections. Here, the shell is discretized using rectangular elements, with element wise thickness treated as an independent design variable.

A square shell is optimized, by minimizing its weight, considering maximum displacement constraint, under different boundary conditions with UDL given in Table 4. The dimensions and material properties of the shell are same as listed in Table 3.

The permissible value for the maximum central displacement of shell under the given loading conditions is taken as 0.3mm. A single objective multi-variable optimization is then carried out using sequential programming approach to bring the optimum solution vector towards the constraint surface. The lower and upper bound of each design variable is kept as h_{lower}=0.3 *mm* and h_{upper}=0.6 *mm* respectively. The thickness distributions obtained for shell under various boundary conditions are shown in Figs. 5-7.The optimized weight and the corresponding maximum displacement for each case is listed in Table 5.

0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.3	0.3	0.3447	0.3446	0.3	0.3	0.3
0.3	0.3	0.3589	0.4155	0.4155	0.3585	0.3	0.3
0.3	0.3	0.3	0.6	0.6	0.3	0.3	0.3
0.3	0.3	0.3	0.6	0.6	0.3	0.3	0.3
0.3	0.3	0.3587	0.4153	0.4153	0.3587	0.3	0.3
0.3	0.3	0.3	0.3445	0.3445	0.3	0.3	0.3
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3

***Figure 5** Element wise thickness distribution of shell under CCCC boundary condition (in mm)*

0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.3	0.3	0.3501	0.3501	0.3	0.3	0.3
0.3	0.3	0.3	0.4509	0.4509	0.3	0.3	0.3
0.3	0.3	0.3	0.5069	0.5069	0.3	0.3	0.3
0.3	0.3	0.3	0.5069	0.5069	0.3	0.3	0.3
0.3	0.3	0.3	0.4506	0.4506	0.3	0.3	0.3
0.3	0.3	0.3	0.3503	0.3503	0.3	0.3	0.3
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3

***Figure 6** Element wise thickness distribution of shell under SFSF boundary condition (in mm)*

0.3034	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3755
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.4929
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.5615
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.5614
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.4927
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3751
0.3035	0.3	0.3	0.3	0.3	0.3	0.3	0.3

***Figure 7** Element wise thickness distribution of shell under CFFF boundary condition (in mm)*

***Table 5:** Optimum weight & maximum deflection of optimized shell under various boundary conditions*

Boundary condition	CCCC	SFSF	CFFF
UDL intensity (*Pa*)	8500	250	75
Maximum deflection (*mm*)	0.3	0.3	0.3
Uniformly optimised thickness (*mm*)	0.4181	0.3628	0.3714
Uniformly optimised weight (*gm*)	11.4612	9.9469	10.1809
Elementwise optimised weight (*gm*)	9.1130	8.8014	8.6809

11. Conclusion

In the present work, the FE formulation for four-noded shell based on degenerated shell theory has been developed to assess the static behavior of singly cylindrical shell structures. A sequential quadratic programming based single objective optimization algorithm has been used to minimize the weight of isotropic shell with element wise variation in thickness as design variables.

From the numerical results it can be observed that the FE results for static deflection of shell are very close to the solutions available in the literature. By using element wise variation in thickness as design variables, there is a substantial increase in the stiffness with reduction in weight of the plate over its uniform thickness counterpart. The thickness profiles of the resulting optimum shell for CCCC & SFSF boundary condition shows a growth in thickness around the centre of the shell. The thickness profile of the optimized shell for CFFF boundary condition shows thicker elements at the free curved edge of shell. The thicknesses of the central elements are slightly thicker compared to the nearby elements. There is symmetry in the thickness distribution about the central lines of the shell.

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